

The Inverse Compton Thermostat in Hot Plasmas Near Accreting Black Holes

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ABSTRACT

The X-ray spectra of accreting black hole systems generally contain components (sometimes dominating the total emission) which are well-fit by thermal Comptonization models with temperatures ~ 100 keV. We demonstrate why, over many orders of magnitude in heating rate and seed photon supply, hot plasmas radiate primarily by inverse Compton scattering, and find equilibrium temperatures within a factor of a few of 100 keV. We also determine quantitatively the (wide) bounds on heating rate and seed photon supply for which this statement is true.

Plasmas in thermal balance in this regime obey two simple scaling laws: $\Theta\tau_T \simeq 0.1(l_h/l_s)^{1/4}$; and $\alpha \simeq 1.6(l_s/l_h)^{1/4}$. Here the hot plasma heating rate compactness is l_h , the seed photon compactness is l_s , the temperature in electron rest mass units is Θ , and the Thomson optical depth is τ_T . The coefficient in the first expression is weakly-dependent on plasma geometry; the second expression is independent of geometry. Only when l_s/l_h is a few tenths or greater is there a weak secondary dependence in both relations on τ_T .

Because α is almost independent of everything but l_s/l_h , the observed power law index may be used to estimate l_s/l_h . In both AGN and stellar black holes, the mean value estimated this way is $l_s/l_h \sim 0.1$, although there is much greater sample dispersion among stellar black holes than among AGN. This inference favors models in which the intrinsic (as opposed to reprocessed) luminosity in soft photons entering the hot plasma is small, or in which the hard X-ray production is comparatively distant from the source of soft photons. In addition, it predicts that $\Theta\tau_T \simeq 0.1 - 0.2$, depending primarily on plasma geometry. It is possible to construct coronal models (*i.e.* models in which $l_s/l_h \simeq 0.5$) which fit the observed spectra, but they are tightly constrained: τ_T must be $\simeq 0.08$ and $\Theta \simeq 0.8$.

1. Introduction

Ever since the paper by Shapiro *et al.* (1976), it has been a commonplace to explain the hard X-ray spectra of accreting black hole systems, whether they are galactic binaries or AGN, by thermal Comptonization. Indeed, such models often give very good descriptions of the broad band X-ray spectra of these systems (Sunyaev & Titarchuk 1980; Grebenev *et al.* 1994; Johnson *et al.* 1994; Zdziarski *et al.* 1994; Haardt *et al.* 1993). In recent years ART/P, SIGMA and OSSE observations have substantially increased the volume of data available for constraining these models, but the basic framework has been sustained: for all systems identified on the basis of independent arguments as

accreting black holes, thermal Comptonization is an acceptable description of the X-ray spectrum from $\simeq 3 - 300$ keV to within the uncertainties in both measurements and theory.

The best fit values for the model’s two free parameters, temperature and optical depth, are almost always in the range $30 - 300$ keV and $0.1 - 5$, respectively (Haardt *et al.* 1993; Bouchet *et al.* 1993; Johnson *et al.* 1993; Zdziarski *et al.* 1994). There is certainly no selection effect that could explain this comparatively narrow range of parameters in the AGN, for most are discovered by optical, ultraviolet, or radio searches. While galactic black hole candidates are almost all discovered by X-ray techniques, these searches would have been sensitive to a much wider range of temperatures: for fixed luminosity and optical depth, many of these sources could easily have been detected if their temperature had been as low as ~ 1 keV or as high as ~ 1 MeV.

In most previous work in this field, X-ray spectra fit with thermal Comptonization models are described in terms of the inferred temperature and optical depth. For many purposes this is a reasonable procedure, for the spectral formation models are most directly framed in terms of these parameters, and they do have a direct physical meaning. However, if we are to understand *why* these conditions are found, we will do better to speak in terms of heating rates, cooling rates, and dynamics. In lower temperature plasmas, studies of how dissipative heating and photoionization balance recombination, bremsstrahlung, and collisional excitation led to the realization that the dissipation rate and ionization parameter determine the temperature and ionization state. In high temperature plasmas, dissipation and Compton recoil or pair production using externally-created high energy photons balance annihilation, bremsstrahlung, and inverse Compton scattering of low energy photons; it is the primary goal of this paper to learn how this balance scales with the externally-imposed parameters and boundary conditions.

A principal result of this program will be to explain why it is that inverse Comptonization dominates other radiation processes in these systems, and why its parameters are restricted to such a comparatively narrow range. We will argue that if one frames the issue in terms of the physically causal parameters, namely the heating rate and the seed photon supply rate, the observed parameters are approximate “fixed points” of the nonlinear equations which determine them. This approach also turns out to have a second important consequence: when attention is restricted to values of temperature and optical depth which are actual equilibria, simple scaling laws emerge relating observables to the causal parameters.

We begin by outlining the physical processes which operate in these systems (§2), and then describe how we compute equilibria (§3). In §4 we show quantitatively over how wide a range of causal parameters inverse Comptonization dominates the cooling rate. We also show that the equilibrium temperature does not depend in any direct way on the traditional Compton y -parameter, but that a different scaling relation does describe the equilibrium quite well. In addition we discover a simple relation between the power-law index and the ratio of soft compactness to hard. We then discuss in §5 observable consequences of the scaling relations, and why the causal parameters for accreting black holes almost always lie within the (large) volume of parameter space corresponding to the narrow range of observed parameters. We summarize in §6.

2. Thermodynamics of Hot Plasmas

To highlight the basic physics of these hot plasmas, we focus on a highly idealized model (following, *e.g.* Björnsson & Svensson 1991, BS91 hereafter): a spherical, homogeneous cloud of pure ionized hydrogen in which all particle distribution functions are (relativistic) Maxwellians at the same temperature $\Theta \equiv kT/m_e c^2$ (see Pietrini & Krolik 1994, PK94 hereafter). This temperature is maintained by a heating rate per unit volume $\dot{H} = 3L_h/(4\pi R^3)$, where R is the radius of the cloud

and L_h is its luminosity. Photons with initial energy $x_0 m_e c^2$ ($x_0 \ll \Theta$) are supplied externally in such a way that the photon intensity—at x_0 and all other photon energies $x m_e c^2$ —is isotropic and constant at every point in the cloud.

Two sorts of equilibria must be established: between pair production and annihilation; and between heating and cooling. Because the cooling rate depends on the number density of leptons, while the rate of pair production depends on the number density of high energy photons, these two equilibria are strongly coupled. Although (approximate) fully self-consistent solutions have been obtained for the version of this problem without an independent source of soft photons (BS91), previous treatments of equilibria in which externally generated soft photons enter the Comptonizing cloud (Zdziarski 1985; Haardt & Maraschi 1991, 1993; Ghisellini & Haardt 1994) have a number of limitations, some of which we remove in the present work. For example, Haardt & Maraschi (1991, 1993) have modeled the production of X-rays in AGNs with a system in which a hot corona of total (*i.e.* electron plus positron) Thomson optical depth $\tau_T < 1$ covers an accretion disk. In their model the entire accretion power of the disk is dissipated in the corona. Because the disk thermalizes and reradiates the majority of the X-rays directed toward it, the ratio between the seed photon luminosity L_s and the corona luminosity L_h is fixed at approximately 0.5. We extend the range of possibilities to include both larger optical depths and other ratios between the seed photon luminosity and the plasma luminosity. In addition, their thermal balance does not include annihilation losses, internal photon production in the corona, or the Comptonization losses on the internally-generated photons. In their limited range of parameters, these approximations are justified, but we wish to study these physical circumstances over a wider range of parameters. On the other hand, Ghisellini & Haardt (1994) cover a much larger range in the luminosity ratio L_s/L_h , but they assume a pure pair plasma, and so are limited to follow a narrow track in heating rate and optical depth.

2.1. Key parameters

Lightman (1982) and Svensson (1982) showed that it is a very good approximation to suppose that pair balance equilibria depend on the net lepton optical depth τ_p rather than on the net lepton density or size of the plasma cloud separately. Note that in the pure hydrogen limit, $\tau_p = n_p \sigma_T R$, where n_p is the proton density.

Because Svensson (1984: S84) also showed that the number of pairs $z = n_+/n_p$, where n_+ is the positron number density, could be found as a function of Θ and τ_p , most treatments since have taken Θ as one of the independent variables of the system. In fact, for a given fixed value of Θ , the pair balance equation, $\dot{n}_+ = \dot{n}_A$ (where \dot{n}_+ and \dot{n}_A are the total pair production rate and the pair annihilation rates, respectively), can be solved to give $z = z(\Theta, \tau_p)$, independently of the thermal balance of the system, *i.e.*, without taking care of how much heating is actually required to maintain this temperature. Moreover, with this approach, one finds that there are either two distinct equilibrium solutions or no solutions at all for a given choice of (Θ, τ_p) , thus implying the existence of two equilibrium “branches” for the equilibrium curve $z = z(\Theta)$ for a given τ_p value (see S84, BS91, PK94). One branch corresponds to solutions for which $z \gtrsim 1$, and is determined essentially by photon (pair-creation) processes; the other branch is controlled (mostly) by particle processes, and $z \ll 1$. In a $z = z(\Theta)$ plot the two branches connect at a maximum allowed value for the plasma temperature, $\Theta_{max}(\tau_p)$; at greater temperatures the pair production rate always exceeds the annihilation rate, so that no equilibrium can be found.

Once the pair balance has been found, for a given Θ , the heating rate \dot{H} (per unit volume) required to maintain that temperature can be determined by equating it to the sum of all the cooling rates, which can be written as functions of Θ , τ_p , and z . In causal terms, however, the temperature is a derived parameter which depends on \dot{H} , therefore we can choose to take \dot{H} as the parameter

and derive consequently the temperature, together with z , by coupling the thermal balance equation to the pair balance one. As pointed out in BS91, the thermal balance equation can be written most concisely in terms of the compactness

$$l_h = \frac{\sigma_T}{m_e c^3} \frac{4\pi R^2}{3} \dot{H}. \quad (1)$$

Thus, for a given value of τ_p , this equation and the pair balance equation may be inverted to find $\Theta(l_h, \tau_p)$ and $z(l_h, \tau_p)$. These solutions are single-valued, and, therefore more useful than the ones in terms of Θ .

Our work differs from that of Svensson in adding a new element, external soft photon input. Two more parameters (or more) are required to specify this. We suppose, for simplicity, that the soft photon spectrum is a δ -function at energy x_0 . We call their (constant) number density n_0 ; the corresponding dimensionless quantity is the ratio n_0/n_p . It is useful to also treat the density of soft photons in terms of the alternative dimensionless quantity, the soft photon compactness,

$$l_s = \frac{\sigma_T}{m_e c^3} \frac{L_s}{R}, \quad (2)$$

where L_s is the luminosity of soft photons necessary to maintain the density n_0 . The actual value of L_s depends on geometry; we assume that it is the luminosity which exactly balances the rate at which soft photons leave the cloud:

$$L_s = 4\pi R^2 \int_{-1}^0 d\mu \mu I_s = 4\pi^2 R^2 I_s, \quad (3)$$

where the intensity of soft photons is given by

$$I_s(x) = \frac{c}{4\pi} m_e c^2 x n_0 \delta(x - x_0). \quad (4)$$

Thus, the soft photon compactness definition (eq. (2)) reduces to

$$l_s = \pi \tau_p x_0 \frac{n_0}{n_p}. \quad (5)$$

In summary, the equilibrium conditions of our system are determined by four key causal parameters:

- 1) cloud (hard) compactness l_h ;
- 2) cloud net lepton (or, equivalently, proton) Thomson optical depth τ_p ;
- 3) external soft photon compactness l_s (or n_0/n_p);
- 4) soft photon energy x_0 .

For a given choice of these, from the solution of the coupled pair and thermal balance equations one derives the other key parameters:

- a) plasma temperature $\Theta = \Theta(l_h, \tau_p, l_s, x_0)$,
- b) pair density, $z \equiv n_+/n_p \Rightarrow$ total optical depth to scattering $\tau_T \equiv \sigma_T(2n_+ + n_p)R = (2z + 1)\tau_p$;

We stress that over the very large volume of parameter space we consider solutions parameterized by l_s/l_h do not depend strongly on x_0 . This is because Comptonization is unsaturated (in the sense that over the great majority of the parameter space of interest, most of the injected photons are not scattered all the way up to the electron temperature), so that the Compton cooling rate depends more directly on the energy density of seed photons than on their number density. Therefore, we can focus our attention on the first three key parameters mentioned above and their interrelations.

2.2. The Compton Parameter y

It has long been conventional to describe the results of Comptonization by a single parameter, which we call

$$y_{nr} = 4\Theta \max(\tau_T, \tau_T^2), \quad (6)$$

where $\max(\tau_T, \tau_T^2)$ is an estimate of the mean number of scatterings a photon suffers before escape, and 4Θ is the the mean amplification factor, that is the average fractional photon energy increase due to a scattering event with an electron drawn from a Maxwellian distribution at non-relativistic temperature Θ (when the electron energy is much larger than that of the photon). This y_{nr} parameter (the subscript nr means non relativistic) arises as the key dimensionless quantity when one writes a Fokker-Planck equation for the evolution of photon energies as a result of Comptonization (Kompaneets 1957; Rybicki & Lightman 1979).

In certain commonly encountered limits, it has several important physical interpretations. In the limit in which $\Theta \ll 1$ and $\tau_T \gg 1$, the shape of the emergent spectrum may be written in terms of y_{nr} (Illarionov & Sunyaev 1975, Sunyaev & Titarchuk 1985). When $\Theta \ll 1$ and $y_{nr} \ll \ln(3\Theta/x_0)$, the average ratio between the photons’ energy on escape and their energy upon entry is simply $\exp(y_{nr})$ (Illarionov & Sunyaev 1975; Shapiro *et al.* 1976). Sometimes (*e.g.* Liang 1979) this is taken to mean that the ratio of the emergent luminosity to the seed luminosity is $\exp(y_{nr})$, but this is not correct. As pointed out by Sunyaev & Titarchuk (1985) and Dermer *et al.* (1991), the output luminosity is usually dominated by photons on the tail of the energy change distribution, not the photons whose energy change is near the mean (see also §4.4.2). It also takes more time than average for these higher energy photons to emerge because they must scatter more times to reach those energies. In addition, when the output spectrum is “hard” enough, the photon energy range which dominates the luminosity approaches the energy Θ , so that saturation effects can be important for them, even if they are not important for the average photon. For all these reasons, $\exp(y_{nr})$ is not a very good expression for the relation between L_h and L_s . In §4.4.3 we derive a much more accurate scaling law.

There are other difficulties which also limit the utility of the y -parameter for the problem we consider. When Θ is not small compared to unity, photon energies do not change by small fractional amounts per scatter, and a Fokker-Planck description is therefore of questionable validity. As for the relation of y with the spectral shape, this turns out to be quite problematic as well; in fact, even when $\tau_T \gtrsim 3$, we often deal with transrelativistic or mildly relativistic temperatures, bringing back the issue just discussed above. On the other hand, we are also interested in cases in which τ_T is not very large, and for values of $\tau_T \sim 1$ or smaller, the diffusion approximation breaks down, and the spectral shape is not directly related to y (see § 3.1.1), even in principle.

Despite these difficulties, several attempts have been made to find generalizations of the y -parameter to relativistic temperatures. All the different attempts agree about how to generalize the expression for the mean photon amplification per scattering to relativistic temperatures:

$$A_r = 1 + 4\Theta \frac{K_3(1/\Theta)}{K_2(1/\Theta)} = 1 + 4\Theta \frac{K_1(1/\Theta)}{K_2(1/\Theta)} + 16\Theta^2, \quad (7)$$

where K_n is the n th-order modified Bessel function, and, following S84, its usual approximation is

$$A_r \simeq 1 + 4\Theta + 16\Theta^2, \quad (8)$$

at least for not too large values of Θ .

However, there is significant disagreement about the best way to generalize the y parameter. Loeb, McKee, & Lahav (1991) make the hypothesis that the probability distribution for the number of scatterings that a photon can suffer is a Poisson distribution. They then find that the ratio of the average energy of the escaping photon to its initial value is, similarly to the non-relativistic case, $\exp(y_{tot})$, where the parameter $y_{tot} = (A_r - 1) < u >$, and $< u >$ is the average number of scatterings

for a photon before escape. Therefore, in this view, the representative relativistic Compton parameter can be defined as $y_r \equiv y_{tot}$, so that, approximating A_r with eq. (8),

$$y_r \simeq (4\Theta + 16\Theta^2)\tau_T \left(1 + \frac{\tau_T}{3}\right), \quad (9)$$

where $\langle u \rangle \simeq \tau_T(1 + \tau_T/3)$ is also a reasonable approximation.

On the other hand, Zdziarski *et al.* (1990) estimate the ratio of escape energy to initial energy is by supposing that every photon escapes after suffering the mean number of scatters. With this approximation, the relativistic Compton parameter is

$$y_* = \ln(1 + 4\Theta + 16\Theta^2)\tau_T(1 + \tau_T/3). \quad (10)$$

where the degree of approximation for A_r is the same as in eq. (9).

In the range of physical conditions for the Comptonizing plasma we are interested in, the two different generalizations discussed above give rather different estimates for the parameter, and, in practice we find that neither of them really directly helps describing even qualitative features of Thermal Comptonization in the mildly relativistic temperature regime, as we discuss in § 4.4.2.

As a matter of fact, in this regime, for even more reasons than those discussed above for the general case, the use of a Compton parameter for a description of the conditions of thermal Comptonization is not much justified. This is probably true for what regards the characterization of the spectral shape as well, for which the relevance of the Compton parameter is actually still an open question (see Titarchuk 1994, and Zdziarski, private communication).

3. Calculation

As we remarked before, we must solve two equilibrium conditions, one for pair balance and one for thermal balance. Here we describe the physical processes we included, the approximations we made, and how we arrived at our solutions.

3.1. Pair balance

We write the pair balance equation as follows:

$$\dot{n}_A = (\dot{n}_+)_{\gamma\gamma} + (\dot{n}_+)_{\gamma e} + (\dot{n}_+)_{\gamma p}, \quad (11)$$

where the annihilation rate per unit volume \dot{n}_A is equated to the sum of the relevant pair production rates, namely photon-photon (indicated by the subscript $\gamma - \gamma$), photon-electron ($\gamma - e$), and photon-proton ($\gamma - p$). Particle-particle pair creation rates are neglected here, since they are always negligible (see PK94 and references therein). We also assume that no pairs escape from the cloud.

All of the pair creation rates included depend on the total Comptonized radiation field in the plasma cloud, which is given by the sum of the contribution due to Comptonization of the internally generated photons, and that coming from Comptonization of the externally created soft photons. For this reason, we must define a representation for both contributions.

3.1.1. Comptonized Radiation Field

Following S84, we approximate the contribution from Comptonization of the photons originating in the cloud plasma with the photon number density spectral distribution

$$n_c(x) = n_W(x) + n_F(x) = \left[\frac{n_\gamma}{2} \frac{x^2}{\Theta^2} \frac{e^{-x/\Theta}}{\Theta} \right] + \left[n_1 \frac{e^{-x/\Theta}}{x} \right]. \quad (12)$$

$n_c(x)$ is the sum of a Wien spectral component, $n_W(x)$, and a “flat” component, $n_F(x)$. The factor n_1 is the flat component normalization, explicitly defined in BS91; n_γ is the number density of Wien photons from Comptonization of internally generated radiation, and it is derived as

$$n_\gamma = t_{esc}(f_B \dot{n}_{Br} + f_{DC} \dot{n}_{DC}), \quad (13)$$

where \dot{n}_{Br} and \dot{n}_{DC} are the energy integrated photon production rates for bremsstrahlung and for Double Compton process respectively, explicitly defined in Svensson (1984) and in PK94; f_B and f_{DC} (again defined in S84) are the probabilities that a soft photon of bremsstrahlung or Double Compton origin is scattered into the Wien peak before escaping the cloud; these also depend on the parameter y_1 defined in S84 which describes the approach to Wien equilibrium for internally-generated photons. Therefore, n_1 and n_γ are functions of Θ and τ_T . The photon escape time from the cloud, t_{esc} , is a useful quantity to give a rough description of radiative transfer effects, and it is here defined as in BS91,

$$t_{esc}(\tau_T, \Theta) = \left(\frac{R}{c} \right) \left(1 + \frac{1}{3} g_\tau \tau_T \right), \quad (14)$$

where $g_\tau(\Theta)\tau_T = \tau_w$ is the scattering optical depth including the Klein-Nishina cross section decline at high energies, by averaging its effects on a Wien photon distribution, and $g_\tau(\Theta)$ is (from BS91)

$$g_\tau(\Theta) = \begin{cases} (1 + 5\Theta + 0.4\Theta^2)^{-1} & \text{for } \Theta \leq 1; \\ (3/16)\Theta^{-2}(\ln(1.12\Theta) + 3/4)[1 + (0.1/\Theta)]^{-1} & \text{for } \Theta \geq 1; \end{cases}$$

the factor 1/3 is related to the spherical geometry chosen (see BS91).

There is another radiation field related relevant quantity that must be determined, and it is the energy x_m below which photons get strongly absorbed; this, in general, represents the lower limit for spectral integrations. As a matter of fact, this energy is defined through a relation which is coupled to the pair and thermal balance (since it depends on Θ and z) and must be actually solved simultaneously with those conditions (see PK94).

Now we consider the contribution to the radiation field inside the cloud due to Comptonization of the soft photons originally created by an external source. Since we are analyzing the problem in the framework of the thermal pair equilibrium model originally studied by S84 and BS91, we can be satisfied with a description of the Comptonized external soft photon contributions to the radiation field which is at the same level of approximation as the one we use for the Comptonized internally generated photons. A representation of this type is given by Zdziarski (1985, hereafter Z85; 1986): the Comptonized “soft photon” spectrum is given, in terms of spectral photon number density $n_{sc}(x)$, as the superposition of a power law with an exponential cutoff at the plasma temperature Θ and a Wien peak at the same temperature,

$$n_{sc}(x) = n_{0p} \left\{ \frac{1}{2} \left(\frac{x}{\Theta} \right)^{-\alpha} \frac{e^{-x/\Theta}}{x} + \frac{1}{2} \left(\frac{n_{0w}}{n_{0p}} \right) \left(\frac{x}{\Theta} \right)^3 \frac{e^{-x/\Theta}}{x} \right\} = n_{pl}(x) + n_{sw}(x). \quad (15)$$

In this relation n_{0w} represents the Wien photon density from Comptonization of external soft photons, and n_{0p} is a normalization factor for the power-law component. The relation that best defines the spectral index α of the power-law component in terms of Θ and τ_T depends on τ_T (Z85, and corrections in Zdziarski 1986):

- For τ_T larger than a few, α is well described by a relativistic generalization (Z85) of the non-relativistic form given by Sunyaev & Titarchuk (1980):

$$\alpha = \left(\frac{9}{4} + \gamma(\tau_T, \Theta) \right)^{1/2} - \frac{3}{2},$$

where $\gamma(\tau_T, \Theta)$ is given by Z85,

$$\gamma(\tau_T, \Theta) = \frac{\pi^2}{3(\tau_T + 2/3)^2 \Theta g_\Theta},$$

with the relativistic correction

$$g_\Theta \equiv K_3(1/\Theta)/K_2(1/\Theta),$$

again given in Z85.

- For small τ_T , the best description is a generalization by Z85 to general Θ and $\tau_T \sim 1$ of the formula originally derived by Pozdnyakov *et al.* (1977) for cases with $\tau_T \ll 1$ and $\Theta \gg 1$:

$$\alpha = -\frac{\ln P_{\tau_T}}{\ln A}, \quad (16)$$

where

$$A = 1 + 4\Theta g_\Theta, \quad (17)$$

and

$$P_{\tau_T} = 1 - \frac{3}{8\tau_T^3} [2\tau_T^2 - 1 + e^{-2\tau_T} (2\tau_T + 1)], \quad (18)$$

(corrected for a misprint, as explained in Z86).

As for the ratio (n_{0w}/n_{0p}) , this is given by Z85 as

$$\frac{n_{0w}}{n_{0p}} = \frac{\Gamma(\alpha)}{\Gamma(2\alpha + 3)} P_{\tau_T}(\tau_T), \quad (19)$$

which is again a generalization of ST80 results.

We have now to relate the normalization factor n_{0p} to the total number density of incoming external soft photons, n_0 . From the conservation of photon number, in the absence of absorption processes, we have that

$$n_0(1 - e^{-\tau_T}) = \int_{x_0}^{\infty} n_{sc}(x) dx = \frac{n_{0p}}{2} \left\{ I_1 + \left(\frac{n_{0w}}{n_{0p}} \right) I_2 \right\}, \quad (20)$$

where

$$I_1 \equiv \int_{x_0/\Theta}^{\infty} t^{-(\alpha+1)} e^{-t} dt$$

and $I_2 \simeq 2$. The lower limit of integration chosen, x_0/Θ , implies that we neglect recoil effects in the Comptonization process. The integral I_1 can be calculated in terms of Whittaker functions, following Gradsteyn & Ryzhik (1980) (integral 3.381.6), and then, since $x_0/\Theta \ll 1$, analytically approximated, by using expressions from Abramowitz & Stegun (1972) (par. 13.1.33, 13.5.5-13.5.12), so that we finally obtain

$$I_1 \simeq \frac{1}{\alpha} \left(\frac{x_0}{\Theta} \right)^{-\alpha} e^{-x_0/\Theta},$$

which is valid for any value of α ($<$ or ≥ 1), and

$$\left(\frac{n_{0w}}{n_p} \right) = \left(\frac{n_0}{n_p} \right) (1 - e^{-\tau_T}) \frac{2}{I_1 + 2(n_{0w}/n_{0p})}. \quad (21)$$

When $\Theta \ll 1$ and $\tau_T \gg 1$ (where the ST80 approximations are valid), these approximations are fairly good fits to the ST80 solutions, particularly at the low and high energy extremes. However, they underestimate the ST80 photon intensity by factors of a few in the neighborhood of $x \sim \Theta$. When $\tau_T \ll 1$, these approximations agree reasonably well with Monte Carlo simulations (Zdziarski 1986). Pair production rates computed on the basis of these approximations should always be fairly accurate because they are most sensitive to the highest energy photons.

3.1.2. Radiative Pair Production Rates

Having made these choices for the representation of the contribution to the radiation field in the cloud due to Comptonized external soft photons, we now describe how we calculated the corresponding contributions to photon-photon and photon-particle pair production rates.

All pair production rates involving photons make use of the Wien component, so our general procedure is to substitute $n_\gamma + n_{0w}$ for n_γ in the appropriate expressions in Appendix B of S84. This is a simple substitution for the photon-particle pair production rates, and for all those contributions to the photon-photon pair creation rate except the interaction of Wien photons with the power-law part of the external photon Comptonized spectrum. For this last part, from the general integral form given by S84, we use the expression

$$(\dot{n}_+)_{wpl} = \frac{cr_e^2\pi}{2}n_{0p}(n_\gamma + n_{0w})\Theta^{\alpha-3}\int_1^\infty \phi(s_0)s_0^{-(\alpha+3)/2}K_{\alpha+3}(2s_0^{1/2}/\Theta)ds_0, \quad (22)$$

where $\phi(s_0)$ is a function given by Gould & Schreder (1967), with corrections from Brown, Mikaelian, & Gould (1973). Thus, the pair creation rate depends on the strength of Comptonization, so that the pair balance, thermal balance, and emergent spectrum calculations must be performed self-consistently.

3.2. Thermal Balance

In a schematic form, the equation of heat balance is

$$\sum_i \Lambda_i = \dot{H}, \quad (23)$$

where each separate cooling mechanism is denoted by Λ_i . The processes we include are:

- a) Bremsstrahlung cooling, Λ_B , which is the sum of the rates for $e-p$, $e-e$, and e^+e^- interactions; explicit expressions are given in BS91. Here we must mention that there is one more mechanism of internal photon production, the double Compton process ($\gamma + e \rightarrow \gamma + \gamma + e$). This can be important in the “low”-temperature range ($\Theta \lesssim 0.1$), in terms of number density of generated photons (S84, PK94). On the other hand, from the energetics point of view, it is negligible with respect to other cooling mechanisms, since the created photons are generally at a much lower energy than the original interacting photon, usually one of the Comptonized high-energy photons (see PK94, Thorne 1981)
- b) Annihilation losses, defined here as $\Lambda_A = \Lambda_a - 2m_e c^2 \dot{n}_A$, where Λ_a is the total annihilation cooling rate; this definition of the cooling rate accounts for the fact that we are interested in the losses to the thermal energy of the plasma, and the rest mass energy loss contribution is actually balanced by pair creation, since pair density is in equilibrium.
- c) Inverse Compton losses on all (*i.e.* bremsstrahlung and, possibly, Double Compton) internally created photons, written as

$$(\Lambda_c)_{br} = m_e c^2 3\Theta n_\gamma / t_{esc},$$

where n_γ is given by eq. (13).

- d) Finally, Inverse Comptonization of the external soft photons, including formation of both the Wien component and the power law component:

- 1) The Compton cooling term due to the contribution of external soft photons to the Wien peak is

$$(\Lambda_c)_{sw} = m_e c^2 3\Theta n_{0w} / t_{esc}, \quad (24)$$

2) The Compton cooling due to the power law portion can be evaluated as

$$(\Lambda_c)_{pl} = \frac{m_e c^2}{t_{esc}} \int_{x_0}^{\infty} n_{pl}(x) x dx = m_e c^2 \frac{n_{0p}}{2t_{esc}} \Theta \int_{x_0/\Theta}^{\infty} t^{-\alpha} e^{-t} dt = m_e c^2 \frac{n_{0p}}{2t_{esc}} \Theta I_0(\alpha, x_0/\Theta), \quad (25)$$

where

$$I_0(\alpha, x_0/\Theta) = \int_{x_0/\Theta}^{\infty} t^{-\alpha} e^{-t} dt \simeq \begin{cases} \Gamma(1-\alpha) & \text{for } 0 < \alpha < 1 \\ (x_0/\Theta)^{-1/2} [|\ln(x_0/\Theta)| - \psi(1)] & \text{for } \alpha = 1 \\ (x_0/\Theta)^{(1-\alpha)/(\alpha-1)} & \text{for } 1 < \alpha < 2 \end{cases}. \quad (26)$$

In this expression $\psi(z)$ is the logarithmic derivative of the gamma function, and $\psi(1) = 0.577156649$.

The escape time t_{esc} is given by eq. (14).

One final correction is necessary: the preceding expressions actually give the total emergent spectrum, which sums the inverse Compton cooling rate with the luminosity injected in soft photons. The net Compton cooling due to processing of the external seed photons is therefore

$$(\Lambda_c)_s = (\Lambda_c)_{sw} + (\Lambda_c)_{pl} - \frac{L_s}{4\pi R^3/3} [1 - \exp(-\tau_T)]. \quad (27)$$

4. Results

4.1. Independence with respect to x_0

In the following subsections all the cases we illustrate were computed with the same initial soft photon energy, namely $x_0 = 10^{-5}$. To check the sensitivity of our results to the choice of x_0 , we have also computed equilibria with x_0 a factor of 10 larger and smaller. On the basis of these results, we can assert that, provided the basic condition $\Theta/x_0 \gg 1$ is fulfilled, for a fixed value of l_s/l_h there is only a very weak dependence of the equilibrium solutions on the actual value of x_0 . For an order of magnitude change in x_0 , the corresponding variations in the equilibrium values of Θ and τ_T typically amount to only a few per cent and are never larger than $\sim 15\%$. We are therefore confident that our results are, indeed, representative of the general problem.

4.2. Pair equilibrium curves: the effects of changing n_0/n_p

We begin by presenting a global view of our equilibria, with an emphasis on illustrating how a variable soft photon supply can affect them. In order to facilitate comparison with previous work (*e.g.* S84, BS91, PK94), in which Θ was generally taken as the independent variable, Fig. 1 displays the normalized pair density $z = n_+/n_p$, the total optical depth τ_T , the required heating rate (as a compactness), and the spectral index α of the output Comptonized power law component of the spectrum, all as functions of Θ . These equilibria share the same $\tau_p (= 1)$ and used our standard value of x_0 , 10^{-5} . In each panel we show a family of curves, parameterized by n_0/n_p . We remind the reader that fixed n_0/n_p is equivalent (for constant $\tau_p x_0$) to fixed l_s .

Increasing n_0/n_p clearly has a number of dramatic effects on the character of the equilibrium:

- a) The maximum temperature, Θ_{max} , that the system can attain steadily decreases; indeed, in almost all cases, the temperature that can be achieved for a given heating rate l_h decreases as l_s increases.
- b) At fixed temperature, the pair content increases on the low- z branch, and decreases on the high- z branch.
- c) As corollaries of the previous point, at fixed temperature on the high- z branch τ_T decreases and α increases, while the behavior is precisely opposite on the low- z branch.
- d) Again for fixed Θ , on the low- z branch increasing n_0/n_p implies an increase in l_h ; however, when $z > 1$, l_h becomes almost independent of n_0/n_p for fixed Θ and the different curves $l_h(\Theta)$ of Fig. 1c all converge.
- e) At very large z (and therefore l_h), the $z(\Theta)$ curves become independent of l_s . The value of l_h at which this happens increases linearly with n_0/n_p ; that is, the curves converge at a single value of l_s/l_h .

We now briefly elaborate on these points in order; most will be re-discussed from the point of view of l_h as the independent parameter in the following subsection, and the physics behind them will then become much clearer.

That the temperature falls as n_0/n_p increases is no surprise; the number of photons available for Compton cooling increases, forcing the temperature to decrease. Note that the *range* of temperatures permitted also narrows as n_0/n_p increases.

Similarly, when there are many soft photons available, fewer electrons are required to account for the total cooling at fixed temperature, so the equilibrium pair content falls on the high- z branch. On the other hand, there is a larger number of photons which may potentially be scattered to energies above the pair production threshold, so the number of pairs on the low- z branch increases.

Fewer pairs translates directly into smaller τ_T , and smaller τ_T implies less Comptonization. Therefore, increasing n_0/n_p spells a softer power-law on the high- z branch, and a (very slightly) harder one on the low- z branch.

The increase in l_h at fixed Θ on the low- z branch noted in point d) is a corollary of point a). Similar reasoning explains a related effect, not illustrated in Fig. 1: increasing τ_p decreases Θ_{max} at fixed n_0/n_p . With a fixed number of seed photons, but greater opportunity to scatter them, the temperature falls. For example, with $n_0/n_p = 10^5$, Θ_{max} falls from 1.12 for $\tau_p = 0.1$ to 0.216 for $\tau_p = 1$.

The insensitivity of the $l_h(\Theta)$ curves to l_s on the high- z branch is due to a combination of different effects. For fixed l_s , the limit of very large l_h produces a condition in which $z \gg 1$, so that τ_T becomes very large and Comptonization is saturated. The output spectrum is then dominated, of course, by the Wien portion. In that limit, the ratio between Wien photons and particles in the plasma is a function only of temperature because pair balance determines the number density of photons. When this is the case l_h also depends only on Θ (S84).

At values of l_h slightly lower than those at which complete Wien equilibrium is attained (i.e., for slightly higher temperatures on the high- z branch), at fixed Θ , larger l_s reduces z and hence τ_T (on the low- z branch, l_s has little influence on τ_T , of course). This in turn leads to a reduction in the amount of energy carried off by the average escaping photon because Comptonization is weakened. However, this reduction is almost exactly compensated by the increase in the number of photons available due to the increase in l_s . We will be able to describe this phenomenon more clearly and quantitatively armed with the formalism and scaling laws developed in §4.4.

However, as Fig. 1 shows, when $l_s \gtrsim 3$, this compensation is no longer effective. In this case, the compactness curve crosses the bundle of curves describing smaller l_s equilibria, and does not join them until true Wien equilibrium is reached at substantially larger values of l_h (see also Fig. 1 in

Svensson 1986 for another illustration of this effect). When l_s is this large, the greater number of photons available for Comptonization becomes more important than the reduction in the mean energy per photon, and a larger heating rate is required to maintain thermal balance. Put another way, more energy is required to approach Wien equilibrium because there are more photons to scatter up to energies $\sim kT$.

Point e) is due to the fact that when $z \gg 1$, the increase in total e^\pm density leads to a large increase in the internal photon production rate by bremsstrahlung. At sufficiently large z , internal photon production dominates over external soft photon input and the Compton cooling on these internally generated photons becomes, in turn, comparable to or dominant over other cooling contributions. At this point the hard compactness is basically determined by the “internal” conditions, i.e., it is (almost) independent of the external photon input. In Fig. 2 we show an example of this behavior, for the case ($\tau_p = 1, n_0/n_p = 10^4$). The various cooling rates Λ_i are plotted in the form of compactnesses:

$$l_i = \frac{4\pi}{3} \tau_p \frac{\Lambda_i}{m_e c^2 n_p (c/R)}, \quad (28)$$

where the subscript i indicates the specific process. As Fig. 2 shows, over most of the range of l_h , inverse Compton cooling on externally-created photons accounts for nearly all the cooling, but at the highest values of l_h , it is overtaken by inverse Compton cooling on internal photons. An inspection of the cooling rates for other cases confirms that the onset of this regime (and the consequent convergence of the $z(\Theta)$ curves) corresponds to the condition in which the compactness ratio l_s/l_h reaches a specific (rather extreme) value, namely $\simeq 10^{-4}$.

4.3. Plasma Temperature as a Function of l_h , l_s/l_h , and τ_p

As we have just seen, discussing our equilibria in the conventional language using Θ as the independent parameter makes many effects difficult to explain because they require arguments based on self-consistency that appear circular. Much greater clarity is achieved by transforming to l_h as independent variable because fundamentally, the temperature is the result of equilibration between a given heating rate and the various cooling mechanisms. Similarly, while τ_T is more closely connected to the degree of Comptonization than τ_p , the extra number of electrons and positrons which changes τ_p to τ_T is a consequence of the pair equilibrium, while τ_p is (we presume) controlled by the forces acting on the plasma. Finally, for the remainder of this paper we will use l_s/l_h as the third independent variable because it is more directly related to the global energy budget (see §4.4) and the geometry of the plasma than is n_0/n_p :

$$\frac{l_s}{l_h} \simeq \left(\frac{L_{s,intr}}{L_h} + C \right) \frac{R^2}{d^2} \phi, \quad (29)$$

where $L_{s,intr}$ is the intrinsic luminosity of the source of soft photons, C is the fraction of solid angle around the hot plasma occupied by optically thick matter capable of absorbing the X-rays and re-emitting their energy in lower energy photons, d is the typical distance from the soft photon source to the hot plasma, and ϕ accounts for possible transfer effects within the hot plasma. Because l_s/l_h has such a simple physical interpretation, it has been widely utilized elsewhere (*e.g.* Haardt & Maraschi 1991, 1993; Ghisellini & Haardt 1994; Dermer *et al.* 1991).

In this section, we present the results of our calculations in terms of these three independent parameters, and describe the trends we find. In the following section we will explain these trends physically. The results we show here span a large volume of parameter space, but with the greatest dynamic range in l_h : $0.03 \leq l_h \leq 3 \times 10^3$; $0.01 \leq l_s/l_h \leq 1$; and $0.1 \leq \tau_p \leq 3$. (We have actually computed equilibria for $l_s/l_h = 0.001$ as well, and the corresponding results are included in the figures referred to in § 4.4.3 and in the related discussion.) This range of parameters should cover most cases relevant to the study of accreting black holes (see §5 for more discussion of this point). We chose

$(\tau_p)_{max} = 3$, because, with increasing l_s , higher values of τ_p would yield temperatures which fall below our range of interest (for example, for $\tau_p = 3$ and $l_s \simeq 0.94$, $\Theta_{max} \simeq 0.11$).

In Figs. 3,4,5 the equilibrium curves as functions of l_h are shown for $l_s/l_h = 1.0, 0.1, 0.01$ respectively; in each figure we plot $\Theta(l_h)$, together with the curves for $\tau_T(l_h)$, $\alpha(l_h)$, and $y_r(l_h)$ (as defined in eq. (9)). Each quantity is also computed for several values of τ_p . In Fig. 3 ($l_s/l_h = 1$), $\tau_p = 0.1, 0.5, 1.0, 2.0$; in Figs. 4 and 5, $\tau_p = 0.1, 0.5, 1.0, 3.0$.

The most prominent feature of these plots is the wide region in l_h over which, taking τ_p and l_s/l_h fixed, Θ is very nearly constant. When $l_s/l_h \gtrsim 0.01$ and $\tau_p > 0.1$, the temperature changes by no more than a few percent over as much as four decades in heating rate!

The existence of this constant temperature regime is not new; it is implicitly contained in the results of Haardt & Maraschi (1991). It appears (see §4.4 for a fuller discussion) whenever inverse Compton scattering of externally supplied photons is the dominant cooling mechanism, l_s/l_h is fixed (as, for example, by absorption and subsequent reradiation in soft photons of a fixed fraction of the hot plasma's radiative output), and there are few pairs, so that $\tau_T \simeq \tau_p$. The new points we make here are to define the range of conditions in which it applies, and to elucidate the physics which controls it.

The actual value of Θ on the flat part of the curve is also rather insensitive to l_s/l_h : an order of magnitude increase in that quantity decreases the temperature by only a factor of 2 – 2.5. It is, however, more sensitive to changes in τ_p : in the range of values we have explored, $\partial \ln \Theta / \partial \ln \tau_p \simeq -1$ if l_h is fixed somewhere in the range for which the temperature is constant (Haardt & Maraschi 1993 find a similar dependence of Θ on τ_p for the specific case of $l_s/l_h \simeq 0.5$).

The dynamic range in l_h corresponding to constant temperature also depends on both the compactness ratio l_s/l_h and τ_p . For fixed l_s/l_h , larger τ_p yields a wider range of l_h in which the temperature is constant. Likewise, for fixed τ_p , increasing l_s/l_h also leads to a widening of the interval in hard compactness over which Θ is constant.

The upper boundary in l_h for the constant temperature range can be anywhere from ~ 1 to $> 3 \times 10^3$. It is set by the point at which pair density starts to be no longer negligible ($z \gtrsim 0.05$), so that τ_T begins to become greater than τ_p , and this is sensitive to both l_s/l_h and τ_p . In those cases where Θ remains flat up to the largest value of l_h shown, that is because z remains very small throughout that range of l_h .

The position of the lower boundary varies rather less than the position of the upper boundary. Throughout our parameter range it is $\sim 0.01 - 0.1$. As we shall show in detail in the following subsection, the lower boundary is set by the point at which other cooling processes (notably bremsstrahlung) become competitive with inverse Compton cooling. The position of this boundary depends hardly at all on l_s/l_h , but does depend somewhat on τ_p : increasing τ_p increases l_h at the lower boundary.

At the very highest values of l_h , all the equilibrium curves corresponding to different values of τ_p , for a given ratio l_s/l_h , tend to coincide, again going towards Wien equilibrium conditions for the plasma. This is clearly seen in Fig. 5, corresponding to the smallest l_s/l_h shown here. This is in fact another manifestation of the independence of Θ with respect to l_h when τ_T is fixed; it is just that when l_h is very large, z is also very large, and τ_T now depends on l_h rather than on τ_p . In fact, if we had plotted these results for fixed τ_T , the constant temperature range would extend to even greater l_h , and the different curves would remain separate even at very large l_h . Note that the larger the value of l_s/l_h , the larger the value of l_h at which this occurs. This is because larger l_s/l_h leads to lower temperature, and consequently, for fixed l_h , a lower rate of pair production.

We close this section by pointing out that, for fixed l_s/l_h , α is independent of l_h over an even broader range of l_h than Θ is; in addition, it is rather less sensitive to τ_T . Several factors contribute to this. In the range of l_h for which Θ is constant, τ_T is also constant, so α , which is a function of

those two variables (§3.1.1), must certainly be constant. However, at larger l_h , where τ_T grows with the greater importance of pairs, Θ falls with increasing l_h in such a way as to cancel the effects of changing τ_T (see §4.4.3). Similarly, the weak dependence on τ_p in the low- z regime is due to the inverse relation of Θ and τ_p (also elaborated more fully in §4.4.3), while the weak dependence on τ_p in the high- z regime is due to the decoupling of τ_T from τ_p when $z \gg 1$. The net result is that, provided l_h is greater than the lower boundary of the constant temperature range, α is very nearly a function of l_s/l_h alone.

4.4. Cooling Physics

4.4.1. Relative importance of specific mechanisms

To explain the trends described in the previous subsection, we now examine the relative importance of the different cooling mechanisms. To illustrate this, we plot the ratios of the different cooling compactnesses, l_i , to the total hard compactness (i.e., the heating rate compactness) l_h as functions of l_h itself for the same values of l_s/l_h and τ_p shown in Figs. 3, 4, and 5. Thus, Figs. 6, 7, and 8 show these plots for $l_s/l_h = 1.0, 0.1, 0.01$ respectively, with each of the four panels corresponding to a given value of τ_p . The curves shown in each plot are: the ratio l_{hc}/l_h of the total Compton cooling compactness to the total compactness; the ratio l_B/l_h of the bremsstrahlung cooling compactness to the total compactness; and the ratio l_A/l_h , where l_A is the annihilation cooling compactness.

It is apparent that Comptonization losses, l_{hc} , including both the contribution due to inverse Compton on the internal bremsstrahlung photons and that of inverse Compton on the externally-supplied soft photons (that is,

$$l_{hc} = \frac{4\pi}{3} \frac{\tau_p}{m_e c^2 n_p (c/R)} [(\Lambda_c)_{br} + (\Lambda_c)_s], \quad (30)$$

with the notations of section 3.2), completely dominate over a large range in total hard compactness. Compton cooling dominates all the way from the range where l_h is so large that pair processes dominate down to the bottom of the constant temperature range.

Ultimately, at sufficiently small l_h , bremsstrahlung overcomes inverse Compton cooling. The value of l_h where this occurs (and therefore the lower boundary to the constant temperature range), $l_{h,min}$, may be estimated simply by comparing the bremsstrahlung cooling rate to the total heating rate; we find

$$l_{h,min} \simeq c_o \tau_p^2 \Theta^q \quad (31)$$

where $c_o \simeq 0.031$ and $q = 1/2$ when electron-proton bremsstrahlung dominates, and $c_o \simeq 0.16$ and $q = 1$ when electron-electron bremsstrahlung dominates. These estimates are confirmed by the more careful calculations shown in Figs. 6, 7, and 8.

Where $l_h < l_{h,min}$, we expect $\Theta \propto l_h^{1/q}$. The curves in Figs. 3, 4 and 5 allow us to qualitatively check this behavior. Because q is itself an increasing function of Θ for the interesting range of temperatures, we expect Θ to increase most steeply with l_h in the bremsstrahlung-dominated regime when Θ is least. That occurs for the larger values of τ_p and l_s/l_h , an expectation which is at least qualitatively borne out in these figures.

We also observe, comparing the cooling rate plots with the corresponding temperature and optical depth equilibrium figures, that l_A becomes greater than l_B at very close to the same place at which z becomes > 1 .

4.4.2. Thermal equilibrium in the inverse Compton regime: the luminosity enhancement factor

We have now identified the range in hard compactness corresponding to conditions in which thermal Comptonization controls the cooling of the system. In this case, we have

$$l_{hc} = l_h,$$

and the considerations discussed in §2.2 should in principle apply. Except at the very highest l_h , wherever inverse Compton cooling dominates, externally-supplied photons dominate the internally-created. In this section we will discuss how one might attempt to analytically define scaling relations between l_s , l_h , Θ , and τ_T in this regime.

When inverse Compton on externally-supplied photons dominates, we can evaluate the hard luminosity, L_h , as

$$L_h = \frac{4\pi R^3}{3} \frac{m_e c^2}{t_{esc}} \int n_{sc}(x) x dx - L_s (1 - e^{-\tau_T}), \quad (32)$$

where the second term is subtracted in order to obtain the net Compton luminosity, and $n_{sc}(x)$ is given by eq. (15). We define

$$\langle x \rangle_1 \equiv \frac{\int n_{sc}(x) x dx}{\int n_{sc}(x) dx}, \quad (33)$$

where the suffix 1 means that, with this definition (see eqs. (15) and (20)), the average over photon energy includes only those photons which scattered at least once. These definitions yield a simple relation between the input and output compactnesses, which is valid when the spectrum of injected photons is much narrower than the output spectrum (an assumption well-satisfied here):

$$l_h/l_s = \left[\frac{4}{3} \frac{\langle \frac{x}{x_0} \rangle_1}{(1 + g_\tau \tau_T/3)} - 1 \right] (1 - e^{-\tau_T}), \quad (34)$$

where $\langle x/x_0 \rangle_1$ is the mean factor by which the energy of incoming photons is amplified before escape, the factor $1 + g_\tau \tau_T/3$ is the ratio between the escape time for the photons and the time R/c , and the factor of $e^{-\tau_T}$ corrects for those soft photons which pass through without scattering at all.

As in Dermer *et al.* (1991), the compactness ratio can be formally expressed in terms of the luminosity enhancement factor, η :

$$L_{hc} = \frac{4\pi R^3}{3} m_e c^2 \int x' \dot{n}_{soft}(x') [\eta(x', \Theta, \tau_T) - 1], \quad (35)$$

where $\dot{n}_{soft}(x')$ is the spectral soft photon injection rate, and η is in general a function of the initial soft photon energy x' , in addition to its dependence on Θ and τ_T . In our case $\dot{n}_{soft}(x') = n_0 \delta(x' - x_0)/(R/c)$. The integrand is multiplied by $\eta - 1$ rather than η so that L_{hc} is the net Compton cooling. Consequently, for the present problem, from eq. (35), we have

$$\frac{L_h}{L_s} = \frac{l_h}{l_s} = \eta(x_0, \Theta, \tau_T) - 1, \quad (36)$$

and, comparing with eq. (34),

$$\eta \equiv e^{-\tau_T} + \frac{4}{3} \frac{\langle \frac{x}{x_0} \rangle_1}{1 + g_\tau \tau_T/3} (1 - e^{-\tau_T}). \quad (37)$$

One commonly followed approach to computing η is to adopt a simple model of inverse Compton scattering in which to reach a given amplification x/x_0 always requires exactly $n(x/x_0)$ scatterings. If $x \ll \Theta$, $n(x/x_0) \simeq \ln(x/x_0)/\ln(A_r)$ (A_r is defined in equation (7)). In terms of this model,

$$\langle \frac{x}{x_0} \rangle = \sum_0^\infty P_n \frac{x}{x_0}(n), \quad (38)$$

where P_n is the probability of escape after exactly n scatterings. Note that the relation between $\langle x/x_0 \rangle$, accounting for those photons that are not scattered at all, and the average we have introduced above (eq. (33)), $\langle x/x_0 \rangle_1$, is simply

$$\left\langle \frac{x}{x_0} \right\rangle = \left\langle \frac{x}{x_0} \right\rangle_1 (1 - e^{-\tau_T}) + e^{-\tau_T}. \quad (39)$$

Several different approximation schemes have been proposed to estimate this sum. The simplest (*e.g.* Zdziarski *et al.* 1990) effectively replaces P_n with δ_{nm} , where δ_{ij} is the Kronecker delta, and m is the mean number of scatterings before escape. Loeb *et al.* (1991) suggested approximating P_n by the Poisson distribution. For the problem at hand, however, P_n is best described by the solution of the “gambler’s ruin” problem (see, *e.g.* Feller 1967). The high- n tail of this distribution falls rather more slowly than in the Poisson distribution. At the same time, the amplification A_r^n increases exponentially with n . Therefore, the largest contribution to the sum in equation (38) is likely to come from a range of n rather larger than m , and the associated amplification can be larger by a very substantial factor. To check just how bad an approximation the Poisson distribution is, we have compared the predictions of the Loeb *et al.* model (using y_r as given by eq. (9)) to the actual values of $\eta(\Theta, \tau_T)$ found in our solutions. The ratio between the two exhibits a very large scatter, with a general offset in the sense that the Poisson model substantially underestimates the true amplification.

For these reasons, an alternative description of Comptonization was proposed by Dermer *et al.* (1991). They basically suggested that the amplification factor $\langle x/x_0 \rangle$ should be computed directly from the output spectrum (which they evaluated with a Monte Carlo simulation). We approximate the output spectrum analytically by a power-law plus a Wien component, so we write

$$\left\langle \frac{x}{x_0} \right\rangle_1 = f_{pl} \frac{\langle x_{pl} \rangle}{x_0} + f_W \frac{\langle x_W \rangle}{x_0} \quad (40)$$

where f_{pl}, f_W are the fractions of the scattered photons going into the power-law and the Wien component, respectively, and $f_{pl} + f_W = 1$. The power-law fraction is

$$f_{pl}(x_0, \Theta, \tau_T) \equiv \frac{\int_{x_0}^{\infty} n_{pl}(x) dx}{n_0(1 - e^{-\tau_T})} = \frac{1}{1 + 2\alpha \frac{\Gamma(\alpha)}{\Gamma(2\alpha+3)} P_{\tau_T}(\tau_T) \left(\frac{x_0}{\Theta}\right)^{\alpha}}, \quad (41)$$

where we have used equations (19) and (26). The average photon energies are

$$\langle x_{pl} \rangle = \begin{cases} x_0 \alpha \left(\frac{\Theta}{x_0}\right)^{1-\alpha} \Gamma(1-\alpha) & \text{for } 0 < \alpha < 1 \\ x_0 \frac{\alpha}{(\alpha-1)} & \text{for } \alpha > 1 \end{cases}, \quad (42)$$

(here we neglect to specify the case for $\alpha = 1$ exactly); and $\langle x_W \rangle = 3\Theta$.

In fact, over almost the entire range of l_s/l_h we have studied, $f_{pl} \simeq 1$, i.e., $f_W \ll f_{pl}$. f_W increases with l_{hc}/l_s , but the greatest value we find for it is $\lesssim 4 \times 10^{-2}$ for $l_{hc}/l_s = 10^3$. At this extreme value, the energy in the Wien term begins to be competitive with the energy in the power-law term; everywhere else it is negligible. Thus, the power-law portion of the inverse Compton losses is nearly always the dominant one.

We therefore make the approximations that $f_{pl} \simeq 1$ and $f_{pl} \langle x_{pl} \rangle \gg f_W \langle x_W \rangle$. In this limit, when $0 < \alpha < 1$ (nearly all our equilibria fall in this range),

$$\frac{l_h}{l_s} \simeq (1 - e^{-\tau_T}) \left[\frac{(4/3)\alpha\Gamma(1-\alpha)(\Theta/x_0)^{1-\alpha}}{1 + g_{\tau_T}\tau_T/3} - 1 \right]. \quad (43)$$

Because α depends in a rather complicated way on Θ and τ_T , there appears to be no simple analytic relation describing the thermal balance in this regime.

On the other hand, when $\alpha > 1$, the analogous relation becomes

$$\frac{l_h}{l_s} \simeq (1 - e^{-\tau_T}) \left[\frac{(4/3)\alpha/(\alpha-1)}{1 + g_{\tau_T}\tau_T/3} - 1 \right].$$

Once again, the absence of a simple functional relation between α and Θ and τ_T prevents further reduction of this form.

4.4.3. Empirical scaling

Our numerical results do, however, reveal several simple scaling laws. Fig. 9 shows Θ as a function of the corresponding equilibrium τ_T , for all our equilibria for which inverse Compton cooling dominates. It is apparent that the points for each value of l_s/l_h describe very nearly a straight line in this diagram, and these lines are all very nearly parallel, with logarithmic slope $\simeq -1$. There is no break apparent in any of these lines, at $\tau_T = 1$ or anywhere else. For fixed τ_p , the spread in τ_T is caused by increasing numbers of pairs. At the same time, differing values of τ_p which share the same l_s/l_h and happen to yield the same τ_T also coincide in Θ , demonstrating that τ_T is the appropriate physical variable for this correlation.

We are thus led to plot, in Fig. 10a, the product $(\Theta\tau_T)$ as a function of the ratio l_s/l_{hc} , for the same choice of equilibrium solutions; Fig. 10b shows the corresponding values for α , again versus l_s/l_{hc} . Again, the relationships are very nearly linear, and with very little scatter. Fitting to the points in Fig. 10a yields the relation

$$l_{hc}/l_s \simeq (10\Theta\tau_T)^\beta, \quad (44)$$

where $\beta = 4 \pm 0.5$. This relationship is a much better description of Comptonization equilibrium than any expression of the form e^y . This empirical scaling law is, of course, what lies behind the trends we described in §4.3.

The physical origin of the correlation with l_s/l_h can be easily seen, at least qualitatively. The cooling rate depends on the seed photon energy density, which rises with increasing l_s/l_h . However, because even unsaturated Comptonization can significantly increase the photon energy density above the energy density directly injected, there is a nonlinear dependence on the two quantities which promote Comptonization, namely Θ and τ_T . That the nonlinear dependence should be describable so simply is surprising, given the arguments of the preceding section, but it does appear to be correct.

This scaling law is not completely new. Haardt & Maraschi (1991) demonstrated that l_s/l_h fixes a function of Θ and τ_T , and in Haardt & Maraschi (1993) they presented results showing that when $\tau_T < 1$, this function depends approximately on the product $\Theta\tau_T$ rather than on the two variables separately (at least for Θ not too large). What is new here is two things: the extension of this relation to $\tau_T > 1$, and the scaling with l_s/l_h .

In addition, we find that the coefficient in this scaling law depends weakly on geometry. For fixed l_s and τ_T , a plane-parallel slab has a higher average soft photon energy density than a sphere because the mean number of scatterings to escape is greater. Greater photon energy density leads to stronger cooling, and hence, for fixed l_h , a lower equilibrium temperature. Consequently, the coefficient of $\Theta\tau_T$ in equation (44) changes from 10 to $\simeq 16$.

We have already remarked (§4.3) that in the inverse Compton dominated regime, α is essentially a function of l_s/l_h alone. In Fig. 10b the plot shows that functional relation. Fitting to these points yields

$$\alpha \simeq 1.6 \left(\frac{l_s}{l_h} \right)^{0.25}. \quad (45)$$

Comparing the scaling laws shown in the two parts of Fig. 10, we find the very simple relation

$$\alpha \simeq \frac{0.16}{\Theta\tau_T}, \quad (46)$$

as can be visually inferred from Figs. 10a, and 10b. The coefficient 0.16 applies in the case of spherical geometry; in slab geometry, it becomes 0.1.

When $l_s/l_h \ll 1$, there is hardly any scatter around the relation given in equation 45; however, when l_s/l_h is more than a few tenths and $\tau_T \leq 1$, there is also a weak secondary dependence on τ_T of the form $\alpha \simeq 1.4\tau_T^{0.12}$. For smaller l_s/l_h or larger τ_T , the spectral index is essentially independent of τ_T .

In slab geometry, the increased number of scatterings for fixed τ_T compensates for the lower temperature, so that the relation between spectral index and l_s/l_h is essentially identical to the spherical case. Slightly more energy is carried off in the Wien component however, to make up for the lower energy cut-off in the power-law component.

The scaling shown in equation 46 is in fact a fairly good approximation to the expression for $\alpha(\Theta, \tau_T)$ given in §3.1.1 in the case of modest optical depth ($1 < \tau_T < 3$) and low temperature ($\Theta \lesssim 0.3$):

$$\alpha \sim \frac{-\ln[1 - 3/(4\tau_T)]}{4\Theta} \sim \frac{3}{16(\tau_T\Theta)}.$$

This expansion predicts the correct scaling and very nearly the correct coefficient (0.19 instead of 0.16). That the scaling law is in fact more broadly applicable is due to the fact that we consider only the portion of the $\Theta - \tau$ plane in which equilibria having l_s/l_h in the range $10^{-3} - 1$ are found. This condition restricts $\Theta\tau$ to be ~ 0.1 , and when this condition is met, equation (46) continues to be a fairly good approximation to the exact expression even when $\tau < 1$. That is, equation (46) only applies when the plasma is in thermal balance.

Just as for the temperature scaling law expressed in equation (44), hints of this relation between spectral index and l_s/l_h have also appeared in the literature. Gil’fanov *et al.* (1994) argued on a qualitative basis that α should depend primarily on l_s/l_h , and vary in the sense that we have found. Working solely on the special case $l_s/l_h \simeq 0.5$, Haardt & Maraschi (1993) computed the spectral index over the range $0.01 < \tau_T < 1$ and found that it hardly varied (the sharp gradient in α they found as τ_T approaches 1 is due to a breakdown in their approximations in this regime). What we have done is to confirm that this relation exists, and to discover its quantitative character.

We have now arrived at a much more powerful description of the scalings between physical parameters in the inverse Compton-dominated regime. It is no accident that they have been found by a systematic approach focusing on causal parameters, rather than fitting to phenomenological parameters. In fact, an approach which treated Θ and τ_T as independent parameters would have been incapable of discovering these correlations because there would have been no sensible way to limit the region of the $\Theta - \tau_T$ plane considered. Only framing the problem with l_h and l_s/l_h as the independent variables reveals these correlations.

5. Consequences for Observable Systems

5.1. The Natural Ranges for l_h , l_s/l_h , and τ_p

5.1.1. l_h

As we have just seen, the temperature is confined to a rather narrow range over a very broad span of l_h and l_s/l_h , provided $0.1 \lesssim \tau_p \lesssim \text{few}$. However, this volume of parameter space, large as it is, is not infinite. We now argue that it would be however, very unlikely for any accreting black hole to be found outside this volume.

First, as many have previously pointed out (*e.g.* Lightman and Zdziarski 1987), if the X-ray-emitting plasma is roughly spherical, l_h may be rewritten in terms of quantities more directly related to the accretion process:

$$l_h = \frac{L_h}{L} \frac{L}{L_E} \frac{2\pi\mu_e/m_e}{r/r_g}, \quad (47)$$

where L/L_E is the total luminosity relative to the Eddington luminosity, $\mu_e \simeq 1.2m_p$ is the mass per electron, and r/r_g is the size of the region relative to the gravitational radius of the central mass. Most of the gravitational energy release in accretion onto a black hole takes place around $5 - 10r_g$, so it would be difficult to make l_h more than 3×10^3 . On the other hand, if the portion of the dissipation going into the X-ray-emitting plasma in that region is at least $\sim 10^{-5}L_E$, l_h is $> 10^{-2}$, the lower limit of the region in which we find almost constant temperature. Thus, if there is enough energy going into the X-ray-emitting plasma near a black hole to make it detectable, l_h will fall into the constant temperature range.

In this respect, coronae near the accretion disks around weakly-magnetized neutron stars should be very similar to those near black holes. Accreting white dwarfs generically should have l_h at least $\sim 10^{-5}$ times smaller for equal mass accretion rate, and are therefore likely to fall just below the constant temperature range of l_h even if $L_h/L \simeq 1$. With $l_h \sim 10^{-4} - 10^{-3}$, coronae in the inner regions of accretion disks around white dwarfs are likely to be in a bremsstrahlung-dominated regime (see Fig. 6) with temperatures of a few keV.

The assumption of roughly spherical geometry is not entirely innocuous. It is almost certainly valid for large l_h , where the plasma is always pair-dominated. When that is true, the electron thermal speeds are comparable to the escape speed even for the smallest possible r/r_g , and the positrons cancel out any restraining electric fields. In addition, the effective Eddington luminosity is reduced by $\sim 10^3$. However, at the small l_h end, where the plasma is not pair-dominated, a less symmetrical geometry (*e.g.* disk-like) can make a difference. For fixed τ_T , the ratio of bremsstrahlung luminosity to inverse Compton luminosity is $\propto (r/h)^2 l_h^{-1}$, where h is the smaller dimension. Thus, flattened geometry can cause the lower bound of the constant temperature range of l_h to increase substantially.

5.1.2. l_s/l_h

If $L_{s,intr}/L_h \ll C$, l_s/l_h is controlled by essentially geometrical factors and is (almost) always ≤ 1 (see equation 29). A common assumption is to suppose that the hot plasma forms a slab bounded on one side by a (comparatively) cool accretion disk (Liang 1979, Haardt & Maraschi 1991). In that case, $l_s/l_h \simeq 0.5$, with some order unity corrections due to anisotropy in the upscattering (Ghisellini *et al.* 1991; Haardt 1993). When the intrinsic soft luminosity is small, while it is very difficult to make $l_s/l_h > 1$, there are many ways it could be $\ll 1$. Both C and r^2/d^2 , for example, must be less than unity, and could be much less. In addition, modest relativistic motion can alter the effective value of l_s/l_h because this ratio should be evaluated in the bulk frame of the hot plasma. If the hot plasma moves away from the soft photon source with speed β , the comoving energy density of soft photons falls as $[\gamma(1 + \beta)]^{-4}$. Motion towards the soft photon source would have the opposite effect, of course, but it would take rather contrived dynamics to produce such a situation.

When $L_{s,intr}$ is not negligible, l_s/l_h can easily be > 1 . However, as shown in §4.4.3, the equilibrium temperature on the flat part of the curve is $\propto (l_s/l_h)^{-1/4}$, so rather large values of l_s/l_h are necessary in order to substantially cool the plasma.

5.1.3. τ_p

As is demonstrated in Figures 3, 4, and 5, the total optical depth τ_T becomes independent of τ_p (and a function only of l_h and l_s/l_h) when l_h is large enough for the number of pairs to be much larger than the net lepton number. Regulation of τ_p is therefore only relevant at smaller l_h . The remainder

of this subsection therefore focusses on the range of parameters in which there are relatively few pairs and $\tau_T \simeq \tau_p$.

So far we have relied on a mental picture in which the hot X-ray-emitting plasma is physically well separated from the cool source of soft photons. If they actually touch each other (which is a statement almost equivalent to $l_s/l_h \sim 1$), we must consider thermal mixing via electron conduction. Good magnetic connectivity is probably a reasonable assumption because the most likely way to actually supply the energy to dissipate in the hot plasma is through the passage of MHD waves from an accretion disk to an adjacent hot corona (Stella and Rosner 1984; Tout and Pringle 1992). If wave-particle scattering does not grossly diminish the hot electron mean free path, it makes little sense in this case to speak of a hot plasma unless it is significantly thicker than a Coulomb mean free path. If it were otherwise, much of the volume occupied by the hot electrons would actually be shared with the cool matter (which would then be evaporating into the hot gas: Balbus and McKee 1982). This restriction places a lower limit on τ_T ,

$$\tau_T \gg \frac{8\pi}{3} \Theta^b / \ln \Lambda, \quad (48)$$

where $\ln \Lambda$ is the usual Coulomb logarithm, and b changes from 2 to 4 as Θ increases from $\ll 1$ to $\gg 1$. If $\Theta > 0.1$, the lower limit on τ_T due to thermal conduction is ~ 0.01 .

When the optical depth becomes large, gradients in the radiation pressure develop. Large forces can be associated with these gradients; the minimum radiation force is larger than the force due to gas pressure gradients when $l_h > 4\pi\Theta$ in spherical geometry. In the presence of gravity, radiation pressure dominance often leads to dynamical instabilities (*e.g.* Arons, Klein, & Lea 1987; Krolik 1979). We speculate that these dynamical instabilities may place an upper limit on the optical depth by causing the hot plasma to break up into smaller pieces whose individual optical depths are no bigger than a few.

5.2. The Connection between Spectral Index and Intrinsic Soft Luminosity or Source Geometry

AGN exhibit a rather small range of power law indices in the X-ray band. Type 1 Seyfert galaxies AGN have intrinsic spectral slopes in the 2 – 20 keV band of $\simeq 0.9$ with very little scatter if the effects of Compton reflection and partially-ionized absorption are factored in (Mushotzky *et al.* 1993). Higher luminosity AGN (*i.e.* quasars) have similar spectral slopes, although possible contributions of reflection and ionized absorbers are harder to determine (Mushotzky *et al.* 1993).

If the thermal Comptonization model is correct, we can infer that l_s/l_h is most likely held stably at ~ 0.1 in all of these AGN. Such a small value of l_s/l_h implies that at least one of the following two conditions applies: both $L_{s,intr}/L_h$ and the covering fraction of any reprocessing surface must be small; or the distance from the source of soft photons must be rather greater than the size of the X-ray emitting region. In type 1 Seyfert galaxies, the frequent evidence for strong Compton reflection suggests $C \simeq 0.5$; where this is the case, $(R/d)^2 \phi$ must be ~ 0.1 . Note that contributions to C from the obscuring torus (Krolik *et al.* 1994; Ghisellini *et al.* 1994) automatically have $(R/d)^2 \ll 1$.

Moreover, because α may be used to infer l_s/l_h , and l_s/l_h fixes $\Theta\tau_T$, a characteristic $\alpha \simeq 0.9$ implies that $\Theta\tau_T \simeq 0.1 - 0.2$. The smaller value obtains when the plasma is a slab, the larger value when it is spherical.

The only possible alternative to these inferences is a very specific version of the coronal model (*e.g.* Haardt & Maraschi 1991, 1993; Zdziarski *et al.* 1994), in which $C \simeq 0.5$ and $R \simeq d$. Because $l_s/l_h \simeq 0.5$ in these models, they fall in the regime in which there is a small secondary dependence

of α on τ_T . For this value of l_s/l_h , the observed spectral slope can be produced if τ_T is somewhat less than 0.1. The uniformity of AGN spectra would then require a rather narrow distribution of τ_T . It follows as a corollary that Θ would also have a narrow distribution, centered on $\simeq 0.8$. Heat conduction (§5.1.3) might pose a physical self-consistency problem for models of this sort.

If l_s/l_h is really ~ 0.1 , because the observed ratio of L_s/L_h is often order unity or somewhat greater, some means must be found to reduce the *local* value of l_s/l_h in the the X-ray production region. For example, it might not be immediately adjacent to the region where the bulk of the ultraviolet luminosity is radiated. Another possibility is that the X-ray region moves away from the disk fast enough that the seed photon intensity is diminished by redshifting. All these schemes are made easier if the intrinsic soft luminosity is minimized.

In stellar black hole binaries, the X-ray spectrum is most simply described as the sum of a quasi-thermal soft component and a hard power-law (Tanaka 1989, White 1994). The soft (*i.e.* $T \simeq 1 - 3$ keV) component is presumably due to dissipation in the accretion disk proper, where the density and optical depth are great enough to approximately thermalize the spectrum. Weakly-magnetized accreting neutron stars should also possess such a soft component, although much of the dissipation may occur in shocks near the surface of the neutron star rather than as a result of gradual viscous dissipation in the disk proper. In addition, accreting neutron stars may have rather weaker hard components than accreting black holes if the disk is interrupted more than a few neutron star radii away from the surface.

The hard power-law seen in accreting black hole systems is generally thought to be the result of thermal Comptonization (Shapiro *et al.* 1976). Spectral indices of this hard component exhibit a greater range than in AGN; examples are known from $\alpha \simeq 0.3$ to $\alpha \simeq 1.5$ (Tanaka 1989; Ballet *et al.* 1994; Gil’fanov *et al.* 1994), but the center of the distribution is fairly close to the average spectral index in AGN. We therefore expect a larger range in l_s/l_h , but the mean inferred l_s/l_h for stellar black holes is also ~ 0.1 . As for AGN, very optically thin coronal models can also produce spectra with $\alpha \simeq 1$, but it may be difficult for them to generate spectra as hard as $\alpha \simeq 0.3$.

5.3. Coupling between Fluctuations in the Luminosity and the Spectrum

The simplest model for X-ray fluctuations one might imagine is one in which the geometry and the relative division of heating between hot plasma and cool gas (*i.e.* the factors which determine l_s/l_h) are fixed. If the column density of the corona is fixed independently, changes in luminosity drive changes in l_h , but not in the other parameters. In the large volume of parameter space in which the temperature varies only slowly with l_h , one would then expect that luminosity fluctuations should raise the general continuum level without altering its spectrum, for Θ and τ_T , which determine the X-ray spectrum, are nearly invariant.

In only one AGN (NGC 4151) are the effects of absorption and Compton reflection weak enough that one can attempt to measure possible variations in the intrinsic continuum shape (Yaqoob 1992). In this object (whose mean spectral index is comparatively small, $\simeq 0.5$), α rises from 0.3 to 0.7 while the 2 – 10 keV flux grows by about a factor of four. In the framework of our model, this behavior would be interpreted as an increase in l_s/l_h from ~ 0.001 to ~ 0.04 . Whether this change in l_s/l_h can be genuinely associated with a change in total luminosity is hard to say because the total power in spectra this hard is dominated by the highest energy photons, and the data which exhibit this correlation say nothing about the flux at the photon energies which dominate the output.

There is a much greater wealth of observations regarding stellar black hole X-ray variability than regarding AGN X-ray variability. In the case of stellar black holes, the amplitude of the hard component often fluctuates by very large amounts: changes of three orders of magnitude have been

seen. Yet despite these very large fluctuations, in most examples the spectral index has remained essentially fixed (Tanaka 1989; Churazov *et al.* 1993; Gil’fanov *et al.* 1994). If the *ratio* of soft flux to hard were constant, this behaviour would, of course, be precisely in keeping with the simplest model one could imagine.

However, the hard and soft components in fact appear to vary quite independently. Therefore, if the power law index of the hard component remains constant despite changes in L_s/L_h , there must be an additional element in the model beyond the simplest case. One possibility is that the main source of soft photons is quite distant from the radiation region for the hard component, so that the soft photons serving as Comptonization seeds are entirely from local reprocessing with constant reprocessing fraction. Another possibility is that the distance between the Comptonizing plasma and the main source of soft photons changes in such a way as to cancel the change in luminosity ratio. For example, if the Comptonizing plasma is heated by dissipation of MHD waves originating in an accretion disk, an increase in the intrinsic soft luminosity of the disk might be accompanied by an increase in the flux of MHD waves into the hotter plasma. As a result, the hot plasma could be pushed farther away from the disk, so that the soft photon density in the Comptonizing region is reduced.

In at least one example (Nova Muscae: Ebisawa *et al.* 1994), the spectral index of the hard component *did* change. In this case, as the soft component faded, the spectral index of the hard component fell from $\simeq 1.5$ to $\simeq 0.7$. At least in a qualitative sense, this follows the general trend of decreasing l_s/l_h leading to harder Comptonized power-laws.

The direct relation between spectral shape and l_s/l_h breaks down on the shortest timescales. Spectral equilibration requires a time $t_{eq} \sim \ln(\Theta/x_o)R/(c\tau_T)$ because the highest energy photons are scattered $\simeq \ln(\Theta/x_o)$ times. On the other hand, l_s/l_h could easily change in a time R/c , which is typically an order of magnitude less than t_{eq} . Therefore, these correlations should not be followed on timescales shorter than t_{eq} .

6. Summary

In this paper we have demonstrated that the prevalence of $\simeq 100$ keV thermal Comptonization spectra in the X-ray emission from accreting black holes is no coincidence. Over a very wide range of dimensionless heating rate (i.e. compactness l_h), and normalized strength of seed photon supply (i.e. l_s/l_h), and a somewhat narrower range of net lepton optical depth (τ_p), thermal Compton scattering dominates the heat balance, and the temperature is nearly always approximately of this magnitude. In fact, for fixed l_s/l_h and τ_p , the temperature is virtually independent of heating rate over a dynamic range in l_h that is generally at least several orders of magnitude.

After quantifying the boundaries of this constant temperature regime, we have also argued that nearly all accreting black hole systems should fall within its (large) volume of parameter space. That l_h and l_s/l_h should fall within that range seems very likely, given what we know of black hole accretion dynamics; that τ_p should be in the range giving this result is less certain, but we have raised several plausible arguments toward this end. The compactness is determined by the nature of accretion into a relativistically deep potential; the seed photon supply follows simply from geometrical considerations; the optical depth may be determined by a combination of thermal conduction and radiation pressure dynamics.

We have also shown that simple scaling laws relate the temperature, total (*i.e.* including pairs) Thomson optical depth, and the seed photon supply l_s/l_h . l_s/l_h (almost) uniquely determines the product $\Theta\tau_T$, and, to roughly the same level of approximation, l_s/l_h alone determines the spectral index α of the power-law segment of the Comptonized spectrum.

On the basis of this relationship between α and l_s/l_h we have argued that l_s/l_h in AGN is most likely to be generically ~ 0.1 . Such a small value of l_s/l_h requires any thermalizing surface (for example, an accretion disk) to be comparatively distant from the X-ray emission region, and suggests that the intrinsic (as opposed to reprocessed) soft photon luminosity is comparatively small. If instead the thermalizing surface is nearby, so that $l_s/l_h = 0.5$ or more, the optical depth of the corona must be rather small, and its value is quite tightly constrained; the coronal temperature is then similarly tightly constrained. In stellar black hole binaries, the observed range of power law indices is larger than in AGN, so a larger range of l_s/l_h is permitted, but this range is also roughly centered on 0.1. When stellar black hole binaries make transitions between “soft” and “hard” states, the slope of the Comptonized power law usually remains constant, even while the relative amplitude of the soft thermal and hard Comptonized components change by orders of magnitude. This fact requires the luminosity in seed photons fed into the Comptonizing plasma to not be simply proportional to the soft luminosity we observe.

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Figure Captions

Figure 1 A global view of our equilibria, seen as functions of Θ for $\tau_p = 1.0$ and $x_0 = 10^{-5}$. The various curves are each for a different fixed value of n_0/n_p (and, correspondingly, of l_s : solid is $n_0/n_p = 0$; dotted is $n_0/n_p = 100$ (i.e., $l_s = 3.14 \times 10^{-3}$); short dash is $n_0/n_p = 10^3$ ($l_s = 3.14 \times 10^{-2}$); long-dash is $n_0/n_p = 10^4$ ($l_s = 0.314$); dot-short-dash is $n_0/n_p = 10^5$ ($l_s = 3.14$); dot-long-dash is $n_0/n_p = 10^6$ ($l_s = 31.4$). In the first three panels l_h increases upwards; in the fourth it is the opposite.

a) The normalized pair density n_+/n_p . Pairs dominate only when the equilibrium is well on the high- l_h side of the temperature maximum, but they can significantly influence the temperature even when $z \sim 0.1$.

b) The total Thomson optical depth τ_T .

c) The heating rate in terms of compactness l_h . The curves for different n_0/n_p converge when they become pair-dominated.

d) The power law index of the Comptonized output spectrum. No curve is plotted for $n_0/n_p = 0$ because this spectral component does not exist in that limit.

Figure 2 The relative importance of different cooling processes as functions of l_h for the specific case $\tau_p = 1$, $n_0/n_p = 10^4$ ($l_s = 0.3$). The solid line is inverse Compton cooling on external photons; the dotted curve is inverse Compton cooling on internally created photons; the dashed curve is annihilation cooling; and the dot-dashed curve is bremsstrahlung. For most of the parameter range, inverse Compton cooling on externally-created photons accounts for nearly all the cooling, but at the highest values of l_h , it is overtaken by inverse Compton cooling on internal photons.

Figure 3abcd The principal physical quantities as functions of l_h for $l_s/l_h = 1$. In this figure and Figs. 4 and 5, we show Θ (panel a), τ_T (panel b), α (panel c), and y_r (panel d) as functions of l_h . In each of Figs. 3, 4, and 5, the solid line corresponds to $\tau_p = 0.1$; the dotted line to $\tau_p = 0.5$; the short dashed line to $\tau_p = 1$; the long dashed line to $\tau_p = 2$; and the dot-dashed line to $\tau_p = 3$.

Figure 4abcd The principal physical quantities as functions of l_h for $l_s/l_h = 0.1$. The different curves correspond to different values of τ_p , as described in the caption to Figure 3.

Figure 5abcd The principal physical quantities as functions of l_h for $l_s/l_h = 0.01$. The different curves correspond to different values of τ_p , as described in the caption to Figure 3.

Figure 6abcd The relative contributions of the different cooling mechanisms for $l_s/l_h = 1$. The solid line is bremsstrahlung (the sum of e-e, e-p and e-e⁺); the broken line is total inverse Compton (on both internally and externally created photons); the dotted line is annihilation cooling. In some cases annihilation cooling is so weak that it accounts for less than 10^{-5} of the total, and its curve does not appear.

a) $\tau_p = 0.1$

b) $\tau_p = 0.5$

c) $\tau_p = 1$

d) $\tau_p = 2$

Figure 7abcd The relative contributions of the different cooling mechanisms for $l_s/l_h = 0.1$. The mechanisms associated with each curve are as described in the caption to Figures 6.

a) $\tau_p = 0.1$

b) $\tau_p = 0.5$

c) $\tau_p = 1$

d) $\tau_p = 3$

Figure 8abcd The relative contributions of the different cooling mechanisms for $l_s/l_h = 0.01$. The mechanisms associated with each curve are as described in the caption to Figures 6.

a) $\tau_p = 0.1$

b) $\tau_p = 0.5$

c) $\tau_p = 1$

d) $\tau_p = 3$

Figure 9 Temperature in the Compton-dominated regime as a function of τ_T for a variety of seed photon supply rates. The symbol coding is: $l_s/l_h = 1$ represented by filled circles; $l_s/l_h = 0.5$ by solid squares; $l_s/l_h = 0.1$ by crosses; $l_s/l_h = 0.01$ by open circles; and $l_s/l_h = 0.001$ by open triangles; the open square represents an equilibrium corresponding to $l_s/l_h = 10$.

Figure 10 The scaling of Comptonization model parameters with l_s/l_h . In these figures we have plotted results representative of all the models we computed.

a) $\Theta\tau_T$ as a function of l_s/l_h . Despite differences in l_h and τ_p , all models fall very close to a straight line in $\log(\Theta\tau_T)$ vs. $\log l_s/l_h$. For fixed τ_p , the spread in τ_T is due to pair creation.

b) α as a function of l_s/l_h . Although three independent parameters are required to specify these models (l_h , l_s , and τ_p), the spectral index of the power-law component of the output spectrum depends almost solely on one parameter, l_s/l_h . When $l_s/l_h \simeq 1$, there is a weak secondary dependence on τ_T .